1 a
2 d
3 a
4 b
5 d
6 a

Part 1, MULTIPLE CHOICE, 5 Points Each

~~After~~ ~~Fri 08/27~~

Ex 1 Sol.

1 Let

$$U = \{2, 4, 6, 8, 10, 12, 14, 16\}.$$
$$A = \{2, 4, 6, 8, 10\}$$
$$B = \{4, 8, 12, 16\}$$

Find $(A \cap B)'$.

(a) $\{12, 16\}$ (b) $\{2, 6, 10\}$ ~~(c)~~ $\{2, 6, 10, 12, 14, 16\}$ (d) $\{14\}$ (e) $\{4, 8\}$

$A = \{2, 4, 6, 8, 10\}$ (4 in B, 8 in B)

$A \cap B = \{4, 8\}$.

$U = \{2, \cancel{4}, 6, \cancel{8}, 10, 12, 14, 16\}$

$(A \cap B)' = \{2, 6, 10, 12, 14, 16\}$.

~~After Fri 08/27~~

2 If R and S are finite subsets of a universal set U, such that

$$n(R') = 20, \quad n(S) = 15, \quad n(S' \cap R') = 5 \text{ and } n(U) = 35,$$

how many elements in $S \cap R$.

(a) 5 (b) 10 (c) 25 ~~(d)~~ 0 (e) 15

IT LOOKS LIKE WE MAY Need De MORGAN'S LAW $(A \cup B)' = A' \cap B'$

AND The IN-EX PRINCIPLE $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

We need to find $n(S \cap R) \underset{\text{(by In-Ex)}}{=} n(S) + n(R) - n(S \cup R)$

I know $\boxed{n(S) = 15}$ FROM info given.

I can figure out $n(R)$ by $n(R) = n(U) - n(R') = 35 - 20 = 15$ $\boxed{n(R) = 15}$

That leaves $n(S \cup R) = n(U) - n((S \cup R)') \underset{\text{DeMorgan}}{=} 35 - n(S' \cap R')$

$\boxed{n(S \cup R) = 35 - 5 = 30}$

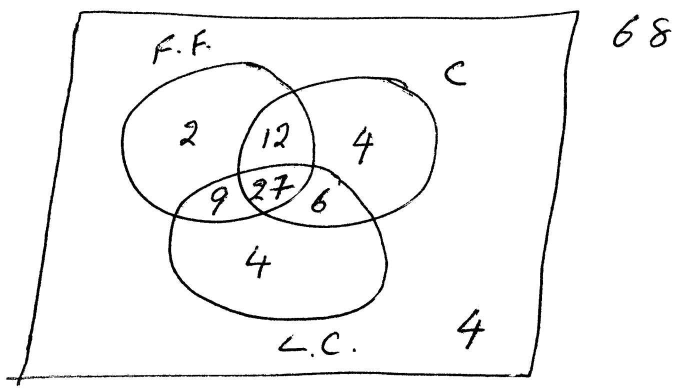
So: $n(S \cap R) = n(S) + n(R) - n(S \cup R) = 15 + 15 - 30 = 0$

~~After Wed 09/01~~

3 A survey of 68 people showed that 50 liked Frosted Flakes, 49 liked Cheerios and 46 liked Lucky Charms. 36 liked both Frosted Flakes and Lucky Charms, 33 liked Cheerios and lucky charms and 39 liked Cheerios and Frosted Flakes. Twenty Seven liked all three types of cereal. How many didn't like any of the three cereals.

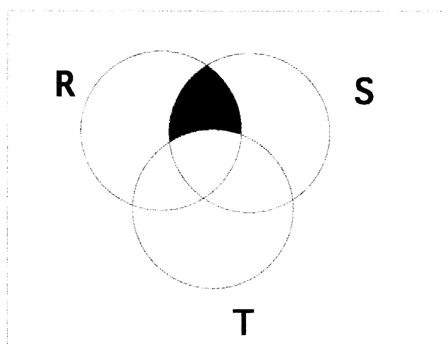
~~(a)~~ 4 (b) 0 (c) 2 (d) 10 (e) 7

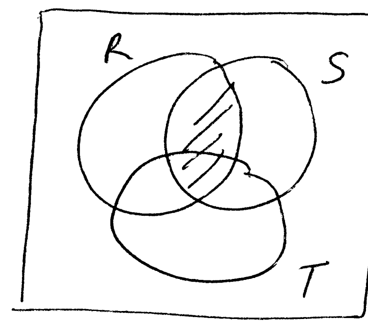
U	68
FF	50
C	49
LC	46
$FF \cap LC$	36
$LC \cap C$	33
$C \cap FF$	39
$C \cap FF \cap LC$	27



~~After Mon 08/30~~

4 Identify the shaded region in the Venn diagram below.





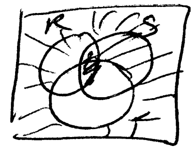
$R \cap S$

(a) $R \cap S \cap T$ ✗ — ↓ in $R \cap S$ and in T

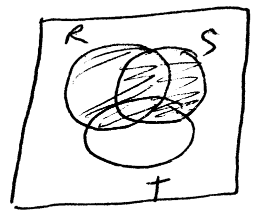
~~(b)~~ $R \cap S \cap T'$ ✓ — ↓ in $R \cap S$ but not in T

(c) $R \cap S$ ✗ — No see Picture

(d) $(R \cap S) \cup T'$ ✗ — in $R \cap S$ or outside T. This includes all of T'



(e) $(R \cap T') \cup S$ ✗ — (in R but not in T) or in S



~~AFTER FRI 09/03~~

5 Five square tiles of the same size but of different colors (all 5 colors are different) are arranged side by side in a horizontal line. How many different patterns are possible?

(*a*) 2^5 (*b*) 5 (*c*) 5^2 (*d*) 120 (*e*) 100

#options $\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120$

~~AFTER MON 09/06~~

6 A chess club consisting of 20 members must choose a president, a secretary and a treasurer. If every club member is eligible for every position and positions cannot be shared, in how many ways can the above three officers be chosen?

(*a*) $P(20,3)$ (*b*) 20^3 (*c*) $C(20,18)$ (*d*) 3^{20} (*e*) $20+19+18$

$\underset{\text{PRES.}}{\underline{20}} \cdot \underset{\text{SEC.}}{\underline{19}} \cdot \underset{\text{TREAS.}}{\underline{18}} = P(20,3)$

~~After~~ ~~Wed Sept 08~~

7 An urn contains 7 numbered balls, 3 red and 4 green. A sample of 3 balls is selected from the urn. How many such samples with 2 red balls an 1 green ball are possible?

(a) $C(3,2)$ (b) $C(7,3)-4$ (c) $\frac{C(7,3)}{2!}$ (d) $C(3,2)\cdot C(4,1)$ (e) $C(4,1)$

3R 4G

Sample size 3

step 1: choose 2R ($C(3,2)$ ways)
step 2: choose 1G ($C(4,1)$ ways)

Samples with 2R and 1G
$= C(3,2)\cdot C(4,1) = 12$

~~After Wed Sept 08~~

8 A poker hand consists of a sample of 5 cards drawn from a deck of 52 cards. How many such hands have exactly three clubs?

Recall that a deck of cards has 13 clubs, 13 hearts, 13 diamonds and 13 spades.

(a) $C(13,3)+C(39,2)$ (b) $3\cdot C(39,2)$ (c) 13^3

(d) $C(13,3)\cdot C(49,2)$ (e) $C(13,3)\cdot C(39,2)$

step 1 choose 3 clubs. $C(13,3)$ ways.
step 2 choose 2 other cards $C(39,2)$

hands with 3 clubs + Two other cards
is $C(13,3)\cdot C(39,2)$

~~After Sept 13~~

9 Which of the following is equal to $C(1000, 850)$?

(a) $C(1000, 150)$ (b) $P(1000, 850)$ (c) $C(1001, 850) + C(1001, 851)$

(d) $P(1001, 850) + P(1001, 851)$ (e) $C(850, 1000)$

$$C(n, R) = C(n, n-R)$$

$$C(1000, 850) = C(1000, 1000-850)$$

$$= C(1000, 150)$$

~~After Sept 10~~

10 In a class of 75 math students we wish to assign each student to one of the projects, Project A (How to lie using statistics), Project B (How to use mathematics to choose the perfect Partner) or Project C (The mathematics of gambling). We will assign 25 students to work on Project A, 30 students to work on Project B and 20 students to work on Project C. In how many ways can the three teams of students be chosen?

(a) $P(75, 25) \cdot P(50, 30) \cdot P(20, 20)$ (b) $C(75, 25) + C(50, 30) + C(20, 20)$

(c) $C(75, 25) \cdot C(75, 30) \cdot C(75, 20)$ (d) $P(75, 25) \cdot P(75, 30) \cdot P(75, 20)$

(e) $C(75, 25) \cdot C(50, 30) \cdot C(20, 20)$

Reverse Urn Problem

75 students

$\xrightarrow{25} A$ $\xrightarrow{30} B$ $\xrightarrow{20} C$

step 1: assign 25 to Proj. A: $C(75, 25)$ ways
step 2: assign 30 to Proj B: $C(50, 30)$ ways
step 3: assign last 20 to Proj C : $C(20, 20)$ ways

ways to assign:

$$C(75, 25) \cdot C(50, 30) \cdot C(20, 20)$$

Part II, PARTIAL CREDIT,

Show all of your work for credit

~~Before Sept~~ 3

11, How many 4 letter words can be made from the letters of the word

THURSDAY $\longrightarrow$ 8 LETTERS

(a) If letters CAN be repeated.

$\underline{8} \cdot \underline{8} \cdot \underline{8} \cdot \underline{8} \cdot \quad = 8^4$

(b) If letters CANNOT be repeated.

$\underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5}$

(c) If the words must end in a vowel and letters CANNOT be repeated (NOTE: y is not a vowel).

$\underline{5} \ \underline{6} \ \underline{7} \cdot \underline{2} \quad = \quad 420$ such words

step 1 assign last letter (must be a U or A)
2 ways

step 2 assign penultimate letter 7 way
step 3 " second letter 6 ways
step 4 " 1st letter 5 ways

12, (a) Use De Morgan's laws to simplify

$$T' \cap (T' \cup S')'$$

$$= T' \cap ((T \cap S)')'$$

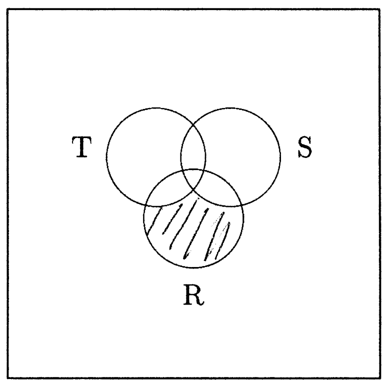
$$= T' \cap (T \cap S) = T' \cap T \cap S = \emptyset \cap S = \emptyset$$

since $T' \cap T = \emptyset$

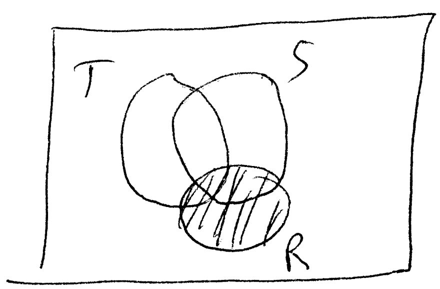
(b) Shade the region corresponding to

$$R \cap (S \cup T)'$$

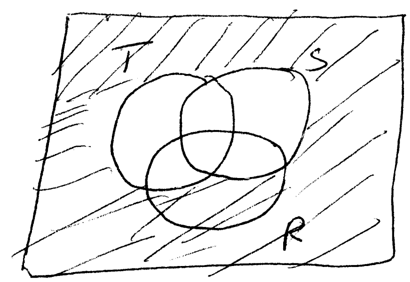
in the diagram below.



$R \cap (S \cup T)'$ = Part shaded in both diagrams below



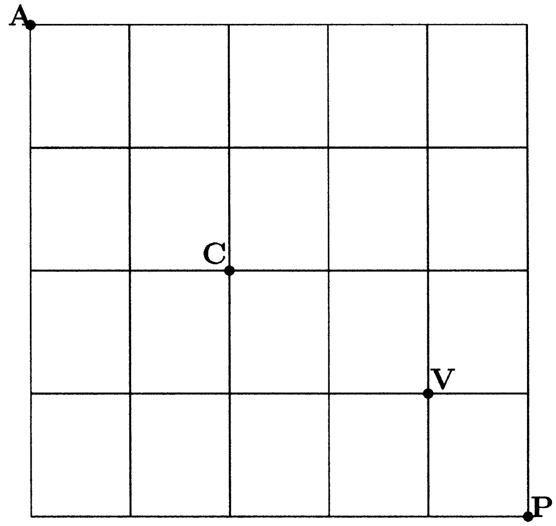
R.



$(S \cup T)'$

~~AFTER Sept 10~~

13 A streetmap of Mathville is given below. You arrive at the Airport at A and wish to take a taxi to Paschal's house at P. The taxi driver, being an honest sort, will take a route from A to P with no backtracking, always travelling south or east.



(a) How many such routes are possible from A to P?

MUST TRAVEL 9 blocks 5 in EASTERLY direction

$C(9,5)$ ways.

(b) If you insist on stopping off at the Combinatorium at C, how many routes can the taxi driver take from A to P? # ROUTES FROM A to P THRU C. $(A \to C \to P)$

= (# ROUTES FROM A to C) · (# Routes from C to P).

$= C(4,2) \cdot C(5,3)$

$= 6 \cdot 10 = 60$

(c) If wish to stop off at both the combinatorium at C and the Vennitarium at V, how many routes can your taxi driver take?

need # Routes of form $A \to C \to V \to P$

= (# Routes A to C) · (# Routes C to V) · (# Routes V to P)

$= C(4,2) \cdot C(3,2) \cdot C(2,1)$

$= 6 \cdot 3 \cdot 2 = 36$

LESS THAN # in Part B because of the extra restriction that you must go through V.

~~After~~ ~~Sept~~

14, The following three yes/no questions were posed to a class of 68 students in a survey:

(i) Do You like Rap music? R = Those who like Rap

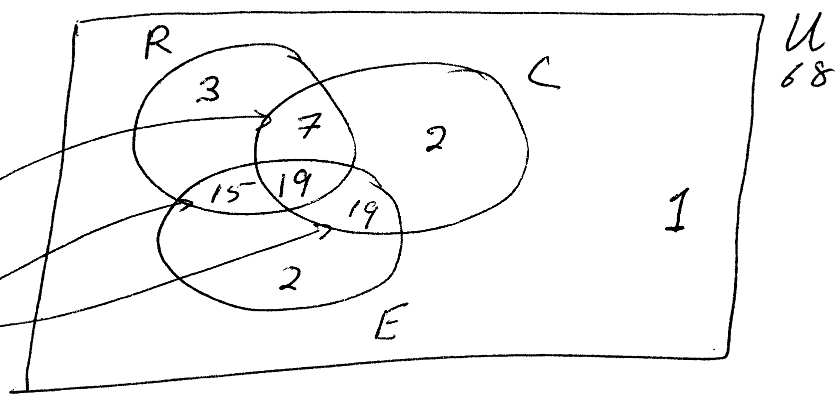
(ii) Do You like Classical music? C = Those who like Classical

(iii) Do You like Eighties music? E = Those who like Eighties music.

The results showed that 44 liked Rap music, 47 liked Classical music and 55 liked Eighties music. Nineteen students liked all three types of music, 7 liked Rap and Classical but not Eighties music, 19 liked Classical and Eighties music, but not Rap and 15 liked Rap and Eighties music, but not Classical.

(a) Present the Data given above on a Venn diagram, where R denotes the set of student's who like Rap, C denotes the set of student's who like Classical and E denotes the set of student's who like Eighties music.

U	68
R	44
C	47
E	55
$R\cap C\cap E$	19
$E'\cap R\cap C$	7
$R'\cap C\cap E$	19
$C'\cap R\cap E$	15



(b) How many students didn't like any of the above music types?

(c) If a student was in the set $E\cap(R\cup C)'$, what answers did they give to questions (i), (ii) and (iii)?

(i) No

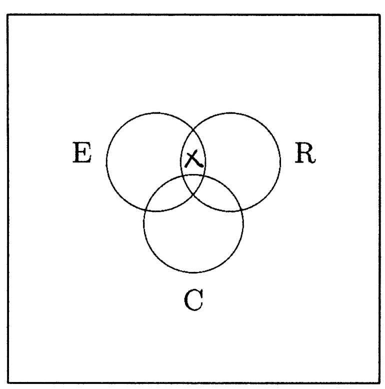
(ii) No

(iii) Yes

$E\cap(R\cup C)' = E\cap R'\cap C'$ = eighties only.

R C E

(c) If a student answered in the following way: (i) yes, (ii) no, (iii) yes, place an X in the following Venn diagram to indicate which basic region corresponds to that student's preferences.



In E

In R

Not in C.

~~After Sept 15~~

15, (a) Fill in the next line of Paschal's triangle below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 + 5 1

1 6 15 20 15 6 1

(b) If you flip a coin 6 times, the result is a sequence of H's and T's of length 6. How many such sequences have at least 3 heads? (Paschal's triangle above may help reduce the amount of work involved)

The # sequences with at least 3 heads

= # sequences with exactly 3 H's

+ # " " " 4 H's

+ # " " " 5 H's

+ # " " " 6 H's

$= C(6,3) + C(6,4) + C(6,5) + C(6,6)$

= sum of the last 4 numbers in the line of Paschal's Triangle that we filled in above

$= 20 + 15 + 6 + 1 = 42$ sequences.